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Luca Pacioli, School Teacher and University Professor

Introduction

This presentation reviews the career of Luca Pacioli from his beginnings as a tutor to the sons of a merchant in the 1460s until his final appointment in Rome in 1514. Pacioli's education and life is briefly described, followed by an overview of education in Northern Italy during the 15th century, first in schools and then in universities. Where Pacioli worked as a teacher is then described. Finally, the transformation of Pacioli's teaching between 1478 and 1494 is described in terms of the impact it had upon the teaching of algebra and its apparent impact upon how he presented his treatise on double entry bookkeeping, *De Scripturis*, in *Summa Arithmetica*.¹

Luca Pacioli received his early education in Sansepolcro. His Perugia manuscript from 1478² and his manuscript book of chess puzzles, *De ludo scachorum* from c.1500³ show that his handwriting was in the calligraphic style known as 'merchant script', or 'mercantesca', suggesting he received an abbaco education – an education for sons of merchants and craftsmen – rather than an education in a grammar school, something that will be returned to later in this presentation and which is consistent with his having been raised as an orphan in Sansepolcro by a merchant, Folco de' Befolci.

At elementary school he would have learnt some Latin – bilingual elementary education was not uncommon at that time⁴ – and he is likely to have developed it further when he was preparing to enter the Friars Minor in the early 1470s, if not before.

¹ Pacioli, L., *Summa de Arithmetica Geometria Proportioni et Proportionalita*, 1494, Paganino de Paganini, Venice.

² Pacioli, L., *Tractatus mathematicus ad discipulos perusinos*, 1478, Vat. Lat. 3129. Manuscript book, Perugia.

³ Pacioli, L., *De Ludo Scachorum*, c.1500, Biblioteca della Fondazione Coronini Cronberg di Gorizia, Manuscript book, Mantova.

⁴ see, for example, Witt, R., What Did Giovannino Read and Write? *Literacy in Early Renaissance Florence. I Tatti Studies: Essays in the Renaissance*, 1995, Vol. 6, pp. 83-114.

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This is important in any consideration of Pacioli's career, and he must have been fluent in Latin long before he compiled *Summa Arithmetica*: teaching in universities at that time was in Latin – “all lectures, disputations, and texts were in Latin.”⁵

Pacioli's first job outside Sansepolcro was for a Venetian merchant, Ser Antonio Rompiasi, in 1464, initially as company for the merchant's three children but, subsequently, as the merchant's assistant. At that time, Pacioli was 16 or 17 years old. He dedicated his first book on arithmetic and algebra to the sons of Rompiasi in 1470,⁶ which suggests that he was more than simply a companion to them. However, given his age, Pacioli would not have acted as their ‘magister’ (teacher) in his early years in Venice. Instead, he most likely acted in the role of a ‘ripetitore’, someone who either assisted a magister in his school or coached wealthy children at home: “[e]ven if a noble father could not afford the exclusive services of a master for his children, he might be able to hire a ripetitore at nominal cost to coach them... Poor university students some times supported themselves this way.”⁷ In support of this hypothesis, it is known that during this period Pacioli attended public lectures at the Scuola di Rialto, a university in all but name.⁸

Another point worth noting concerning Rompiasi and relevant to Pacioli's writing is that Rompiasi was most likely also a notary: the title ‘ser’ being ascribed at that time to notaries and clerics⁹.¹⁰ Notaries had to be skilled in Latin and being a notary was the most common qualification of teachers of grammar at that time¹¹ and, even if he did not practice as a teacher, it is quite possible that Rompiasi helped Pacioli to improve his Latin beyond the level typical of an abbaco education.

⁵ D'Elia, A.F., The Renaissance University, *International Journal of the Classical Tradition*, 2005, Vol. 12, No. 2, pp. 269-277 (p.272).

⁶ Baldi, B., Fra Luca dal Borgo S. Sepolcro, 1589, c.180r. Republished in Boncompagni, B. (ed.), *Bulletino di bibliographia e di storia delle scienze matematiche e fisiche, Volume XII*, 1879, pp. 421-427; Antinori, C., Introduzione. *Luca Pacioli: De Divina Proportione*. Roma: Istituto Poligrafico e Zecca Dello Stato, 2000, p. 11.

⁷ Grendler, P.F., The Organization of Primary and Secondary Education in the Italian Renaissance. *The Catholic Historical Review*, 1985, Vol. 71, No. 2, pp. 185-205 (p. 198).

⁸ Although there was no university in Venice at that time, the City had pretensions to having its own university and the teaching where Pacioli was appointed (the Scuola di Rialto) was of the topics that would be taught in universities – philosophy, logic, and theology, including algebra and arithmetic [Ross, J. T., Venetian Schools and Teachers Fourteenth to Early Sixteenth Century: A Survey and a Study of Giovanni Battista Egnazio, 1976, *Renaissance Quarterly*, 29(4), Winter, pp. 521–566.]

⁹ Rompiasi lived in the Jewish sector of Venice, the Giudecca, and so is not likely to have been a cleric.

¹⁰ Witt, *ibid.*, footnote 33 pp. 92-3.

¹¹ Witt, *ibid.*, footnote 20, pp. 89-90.

During his period in Venice, if not before, Pacioli became acquainted with the leading humanist of his day, Leon Battista Alberti (1404-1472), and became so well acquainted that upon leaving the employ of Ser Antonio Rompiasi in 1470, he spent some time with Alberti in Rome before returning to Sansepolcro. Alberti was one of the leaders of the humanist education movement and promoted the use of the vernacular in order that the maximum number of people could be reached, something Pacioli did consistently in his educational writing.

Many have constructed a plausible picture of the young Pacioli being taught in Sansepolcro by his kinsman Piero dela Francesca.¹² This would be consistent with Pacioli having received an abbaco education, for Piero himself wrote an abbaco text from which Pacioli inserted many problems on regular bodies into *Summa Arithmetica*. As an abbaco text, Peiro's *Trattato d'abaco* was neither a work of the grammar school system, nor a work intended for use in universities.

Before and in-between his duties as a Franciscan friar, for 50 years, Pacioli was a peripatetic teacher across much of Northern Italy. He was one of a rare breed: someone who taught at both school and university level¹³. He switched back and forth. He also taught in the workshops of artists and taught painters, architects, and stonemasons,¹⁴ tutored Leonardo da Vinci and Albrecht Dürer,¹⁵ who may have added his own image to the famous painting of Pacioli attributed to Jacopo de' Barbari (who is best known for his acclaimed map of Venice of 1500¹⁶), perhaps to encourage German patronage of Pacioli's *De Divina Proportione*, published in 1509.

It seems that Pacioli was not concerned with the level at which he taught but, rather, he was motivated to bring improvement in the knowledge and understanding to those who crossed his path, whatever the level.

So, what was this educational system where teachers could switch from school to university and back again?

¹² *see*, for example, Antinori, *ibid.*, p. 11.

¹³ Grendler, P.F., *Schooling in Renaissance Italy: Literacy and Learning, 1300- 1600*. Baltimore: The Johns Hopkins University Press, 1989, (p. 28) only identifies two other teachers who taught at both levels.

¹⁴ Camerota, F., Teaching Euclid in a practical context: Linear perspective and practical geometry. *Science & Education*, 2006, Vol. 15 (2-4): pp. 323–334 (p. 327).

¹⁵ Mackinnon, N., The Portrait of Fra Luca Pacioli, *Mathematical Gazette*, 1993, Vol. 77, pp. 130-219; Galitzia, S. F. in conversation, Firenze, June 19, 2011.

¹⁶ *See*, Falchetta, P. *Jacopo de' Barbari e le vedute di Venezia*, Il Tridente/Marsilio Editori, 1997. [Accessed at www.tridente.it on 30 May 2011]

Education in 15th century Northern Italy

Schools

The existing tradition in Northern Italy of “*scholastic education was fundamentally antipathetic to the empirical study of nature or to any real content in education; texts, not genuine subjects in themselves, were the objects of learning.*”¹⁷ Then, in the 15th century, a new form of school emerged, one that served the same audience as that served by the scholastic grammar schools, the umanisti schools:

The umanisti school was “*an educator of man, capable of shaping a child’s moral character so as not to be preconditioned but free, open in the future to every possible specialization, but before all else humane and whole, with social links to all mankind and endowed with the prerequisites for the mastery of all techniques.*”¹⁸

The change this brought about can be illustrated by reference to how Latin was taught in these two forms of grammar school, typically by teachers qualified as notaries.¹⁹ Pupils in the scholastic grammar schools between the 13th and 15th centuries learnt Latin in two stages. Firstly according to the word order and syntax of modern Romance languages – medieval Latin or ‘*ordo naturalis*’. Then, they progressed to the development of an ornamented prose style, ‘*ordo artificialis*’.

The umanisti schools continued to teach Latin in two stages, *ordo naturalis* followed by study of stylistic or rhetorical manuals, but the second stage was less concerned with ornamented prose and more directly focused upon *Ciceronianism* – a movement to standardize Latin diction by modelling all prose on the writings of Cicero – and used different manuals from those typically used in the scholastic grammar schools. (Black 2001, p. 9)

By the mid-15th century, there were approximately the same number of scholastic schools as there were umanisti schools. In addition, another group of schools had emerged based, many believe, around the early 13th century books, *Liber Abaci* and *Practica Geometriae* of Leonardo Pisano, schools of secular learning.²⁰ These vocational ‘*abbaco*’ schools were for a different audience – the sons of merchants and craftsmen – and were viewed as providing a practical training for a life as a merchant or as a craftsman; and were also

¹⁷ Black, R., Italian Renaissance Education: Changing Perspectives and Continuing Controversies, *Journal of the History of Ideas*, 1991, Vol. 52, No. 2., pp. 315-334 (p. 317).

¹⁸ Garin, E., Guarino Veronese e la cultura a Ferrara, *Ritratti di umanisti*, 1967, pp. 69-106 (p. 75).

¹⁹ Witt, 1995, *ibid.*, p. 89.

²⁰ Black, R., *Humanism and education in medieval and Renaissance Italy*. 2001, Cambridge: Cambridge University Press.

sometimes attended by boys of noble families who wanted to pursue a profession for which such training would be useful.²¹ Approximately the same number of pupils attended these schools as attended the scholastic and umanisti schools combined.²² Compared to the high incidence of notaries teaching in the grammar schools, many of the teachers in the abbaco schools were *mensuratores* ('measurers'²³ in effect, surveyors, architects), mathematicians by training.

The abacco schools were initially funded by merchants and local communities. Pupils learnt some medieval Latin but only equivalent to the first of the two stages taught in the grammar schools and their texts and all teaching were otherwise in the vernacular. Other than in Florence, after an elementary level education typically including reading, writing, business correspondence, and notarial formulas, they progressed at the age of about 11 to secondary level where the syllabus predominantly focused upon business, with sufficient applied mathematics to provide an education for other trades and crafts, such as masons, architects, and artists, after which apprenticeship in a chosen trade began.²⁴ As a result of this education, *mathematical knowledge was by no means an exclusive guild 'secret,' but rather a common educational tool available to all those who had completed a scuola dell' abacco.*²⁵

In Florence, abacco school education was more brief than elsewhere, though the focus was similar: after elementary school, *"parents were faced with a choice [between sending their sons to an abacco school or a] grammar school. In the first, which lasted approximately two years, students learned the mathematical and accounting skills necessary for entering commerce. In the second, students studied Latin literature for up to four or five years, primarily with an eye to a learned vocation."*²⁶

The material taught in the abbaco schools was contained in books written mainly by the teachers and much was borrowed from earlier similar works. The

²¹ Ulivi, E., *Benedetto da Firenze (1429-1479) un maestro d'abaco del XV secolo: Con documenti inediti e con un'Appendice su abacisti e scuole d'abaco a Firenze nei secoli XIII-XVI*. Series: Bollettino di Storia delle Scienze Matematiche, 2002, Vol. 22. Roma: Istituti Editoriali e Poligrafici Internazionali. p. 11.

²² Grendler, 1989, *ibid.*

²³ Pisa, for example, had *mensuratores* as early as 1164: [Accessed at www.tridente.it/venetie/essays/essay05/note01_5.htm on 30 May 2011]

²⁴ Grendler, 1989, *ibid.*

²⁵ Zervas D. F. (1975). The Trattato dell'Abbaco and Andrea Pisano's Design for the Florentine Baptistery Door, *Renaissance Quarterly*, 1975, Vol. 28, No. 4, Studies in the Renaissance Issue, pp. 483-503 (p. 485).

²⁶ Witt, 1995, *ibid.*, p. 84.

mathematical historian Albrecht Heeffer believes that only the chapters in these texts on tables, operations on fractions, monetary units, problems of exchange, and applications of the rule of three were included in the *abbaco* curriculum and that the *abbaco* texts which contained other material were written to show off the author's skills in the art to other *abbaco* masters.

Elisabetta Ulivi goes further in her study of the *abacco* schools of Florence, suggesting that these texts were principally intended as aide memoires for those who had already studied in a *scuola d'abaco* and that, while most of the authors were *abacco* teachers, some were written by merchants, artists and lovers of mathematics.²⁷

The mathematical historian, Warren Van Egmond suggested in 1980²⁸ that these texts reveal a curriculum organised into six principle subject groups that presents a view of a school syllabus firmly founded in applied mathematics covering the topics which varied according to who was teaching it but, in general, pupils of these schools studied a syllabus that ranged from arithmetic to trigonometry, and geometry,²⁹ all from a perspective of their application to business, trade and craft. Adopting a partial focus upon recreational mathematics, techniques such as the Rule of 3 were learnt that could then be applied to solve business problems.

Above all, these *abbaco* schools were intended to provide vocational instruction and the knowledge taught was “*of a decidedly practical and down-to-earth nature... the vast majority of this [material was] not presented theoretically but rather in the form of practical problems to be solved by the student.*”³⁰

Most *abbaco* teachers were self-employed tutors working in their own single-teacher schools³¹ rather than employed in schools with more than one teacher, though some were; and, as with personal tutors today, tutors taught the subjects they wished their pupils to learn. Bookkeeping and the Ledger were taught in some of these schools³² from the early 14th century, if not before; but, if the number of extant texts from the 14th and 15th centuries containing material on these topics is any guide (only a handful including Benedetto Cotrugli's 1458 manuscript

²⁷ Ulivi, 2002, *ibid.*

²⁸ Van Egmond, W., *Practical Mathematics in the Italian Renaissance: a Catalog of Italian Abacus Manuscripts and Printed Books to 1600*, 1981, Firenze: Editrice Giunti Barbèra.

²⁹ Van Egmond included algebra in his list but scholars now believe that algebra was not taught until the mid-16th century.

³⁰ Zervas, 1975, *ibid.* pp. 486-7.

³¹ Grendler, P. F., What Zuanne Read in School: Vernacular Texts in Sixteenth Century Venetian Schools, *The Sixteenth Century Journal*, 1982, Vol. 13, No. 1, pp. 41-54 (p. 43).

³² Grendler, 1989, *ibid.* p. 319.

book,³³ predate Pacioli's *de Scripturis*) not many schools included these subjects in their curriculum.

Universities

Universities in 15th century Italy, as in the rest of Europe, were in their infancy. To be considered a university, an institution of learning had to offer advanced instruction in law, arts, and medicine and have a minimum of six to eight professors teaching civil law, canon law, medicine, logic, natural philosophy and, usually, rhetoric in regular classes at an advanced level. They awarded doctorates and masters degrees, not undergraduate degrees. Their students were typically aged between 18 and 25. The professors delivered 'public' lectures, which were open to anyone to attend and were usually paid by the civil government (commune), though some charged the students fees as well. Before an institution could be considered a university, its commune had to possess a papal or imperial charter which authorised the conferral of licenses (*licentiae docendi* – licences to teach) and masters and doctoral degrees bearing the university's name.³⁴

Mathematics was limited to one or two professors where it was taught. It was considered a part of natural philosophy and linked through astrology to medicine. In Bologna in the early 15th century, teaching of mathematics (then entitled, "astrology") started with Euclidean arithmetic and geometry plus astronomy in first year, astrology was added in the second and third years, and the fourth year was spent studying astronomy and astrology. When astrology ceased to have importance, the name changed to "astronomy" before finally settling on "mathematics" by the mid-16th century. This was the pattern in most of the Italian universities where mathematics was taught.³⁵

All Italian university towns had faculties of theology, but local mendicant order friars, especially Dominicans and Franciscans, lacking links to the university for arts, medicine, and law, did most of the theology teaching in their own monasteries, which also provided almost all of the students.

The first university in Italy was founded in Bologna in the 12th century. Padua

³³ Cotrugli, B., *Il Libro dell' Arte di Mercatura*, 1458, Manuscript book, Napoli. In Tucci, U., Benedetto Cotrugli, Raguseo: *Il Libro dell' Arte di Mercatura*, 1990, Venezia: Arsenale Editrice: pp. 129-255.

³⁴ Grendler, P.F., *The Universities of the Italian Renaissance*. Baltimore: The Johns Hopkins University Press, 2002, pp. 3-5.

³⁵ Grendler, 2002, *ibid.*, pp. 422-429.

was founded in 1222 (becoming the university for Venice when the city was conquered by the Venetian Republic in 1405), followed between 1343 and 1445 by universities in Naples, Siena, [Parma (which operated briefly between 1361 and 1387³⁶), Rome, and Perugia; and then Pisa, Florence, Pavia (the university for Milan), Turin, Ferrara, and Catania. The next was not founded until the late 16th century.³⁷

These were, therefore, the 12 Italian universities in which Pacioli could have worked. In addition, there were six ‘incomplete’ universities in Arezzo, Modena, Piacenza, the two Venetian schools of the Rialto (where Pacioli himself attended public lectures in the 1460s) and San Marco, and the Casa della Sapienza in Pistoia. All offered university level instruction but none had sufficient professors to be recognised as ‘universities’. There were also four ‘universities’ which did not offer advanced instruction: Genoa, Lucca, Urbino, and the College of Physicians in Venice.³⁸

Where Pacioli worked as a teacher

Teachers employed to work in *abbaco* schools were often paid by the commune, through the local university.³⁹ As a result, it is difficult and often impossible to detect from surviving records whether teachers were employed to teach in a school or in a university. However, in Pacioli’s case, there are surviving documents, such as municipal records from Perugia and Florence which Baldassarre Boncompagni found and published in 1869⁴⁰ that indicate the level at which he was teaching. In addition, the limited number of universities in Italy during Pacioli’s lifetime helps to eliminate the possibility that he was employed to teach in a university in some instances.

In the early stages of his career, Pacioli did not have a masters degree or a doctorate. It took four years to obtain one and, while he did attend public lectures at the Scuola di Rialto in Venice in the 1460s, it was an ‘incomplete’ university and

³⁶ Grendler, 2002, *ibid.*, pp. 126-7

³⁷ Grendler, 2002, *ibid.*, p. 2; D’Elia, 2005, *ibid.*, p. 271.

³⁸ Davies, J., Review. *Renaissance Quarterly*, 2003, Vol. 56, No. 3, pp. 757-758 (p. 757).

³⁹ Grendler, 1989, *ibid.*

⁴⁰ Boncompagni, B. (1879). *Intorno alle vite inedite di tre matematici Appendice di documenti inediti relativi a Fra Luca Pacioli*, *Bullettino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche*, 1879, Anno XXII, pp. 432-438.

unable to award degrees⁴¹ and, therefore, *licentiae docendi*, which were needed to have the right to teach in universities. Pacioli himself never indicated he did more than attend public lectures under Domenico Bragadino at the Scuola di Rialto.

Pacioli's first biographer, Bernardino suggested in 1589 that Pacioli became a friar soon after leaving Venice in 1470 but that his mathematical studies delayed his obtaining his master of theology degree.⁴² Elisabetta Ulivi recounted in 2009 how Pacioli was seen in Sansepolcro dressed as a friar on February 26, 1471, and again on August 20, 1472 and August 11, 1473.⁴³ Pacioli would almost certainly have still have been a novitiate in 1471: it was necessary to spend between one and three years in that position before ordination. But, after spending the necessary time as a novitiate he would have been ordained as a friar by the time he left Sansepolcro for Perugia in 1475.

It has been suggested that Pacioli spent some time in Naples in 1472 first working as a merchant and then teaching privately.⁴⁴ However, the former is impossible if he had been ordained by then. The latter is, however, possible and he may have been in Naples teaching *abbaco*⁴⁵ at some point in that year either before or after he is known to have been in Sansepolcro that August.

From 1475 to 1477, he was teaching at the *abbaco* level in Perugia. The municipal records of the city reveal that he was then appointed as a teacher of *abbaco* from 1477 to 1480.⁴⁶ In 1481, he was in Zara (Croatia) and wrote a book on more advanced algebra while there, but there is not evidence that he was also teaching.

It is believed that between mid-1480 and mid-1484 he obtained his degree in theology: on September 24, 1484 he was mentioned as having the title 'Magister Professor'.⁴⁷ Despite his then being qualified to teach in universities, when he returned to Perugia in November 1486 his appointment was again as a teacher of *abbaco* and continued until April 1488. When he returned to Perugia in November 1510, it was again as a teacher of *abbaco*.

⁴¹ Ross, 1976, *ibid.*, p. 529.

⁴² Baldi, B., Fra Luca dal Borgo S. Sepolcro, 1589, c. 180r. Republished in Boncompagni, B. (ed.), *Bullettino di bibliographia e di storia delle scienze matematiche e fisiche*, 1879, Volume XII, pp. 421-427.

⁴³ Ulivi, E., Nuovi documenti su Luca Pacioli. In Giusti, E. and Martelli, M. (eds.) *Pacioli 500 Anni Dopo*. Sansepolcro: Centro Studi "Mario Pancrazi", 2009, pp. 19-58.

⁴⁴ Taylor, R. E., *No Royal Road*. Chapel Hill, NC: University of North Carolina Press, 1942.

⁴⁵ He had not yet obtained his degree in 1472 and so could not have been teaching in the university.

⁴⁶ Boncompagni, 1879, *ibid.* pp. 432-437.

⁴⁷ Ulivi, 2009, *ibid.*, p. 35.

Thus, throughout the three periods when he worked in Perugia (between 1475 and 1480, between 1486 and 1488, and in 1510-11), he was consistently appointed as a teacher of abbaco. The records relating to his appointment in 1510, reveal that he asked to be appointed to teach abbaco and geometry in school: “*During that year he will offer classes (keep schools) in the art of geometry and abbaco with no other salary as others before him and he himself had done and intends to do.*”⁴⁸

At this point, we are faced with a mixture of opinion. Albrecht Heeffer does not believe that Pacioli ever taught abacco after the 1460s. Mario Biagioli takes a different view: that Pacioli was only a teacher of abbaco (a theory that his very low salary in Perugia certainly supports). However, he believes that the University of Perugia created a chair, *ad docendum aritmeticham seu abicum et geometrium*, before 1412 and that Pacioli was appointed to that chair in 1486, to teach ‘arithmetics’ (abbaco).⁴⁹ Paul Grendler accepts that maestri d’abbaco were often paid through universities but believes that abbaco was not taught in universities.⁵⁰ The Perugian Commune records show that Pacioli was paid by the Commune; they do not mention his working in the University; but, they do appear to mention “schools” in his appointments in 1477 and 1510.

Which interpretation is more plausible? I am convinced more by the views of Grendler than those of Biagioli, who was writing before Grendler but whose work Grendler does not cite in his book on Italian universities of the Renaissance published in 2002 which, in itself, is not remarkable – it was not as easy in the late 20th century to find articles on this or any topic as it is today. Biagioli himself states that his lists of teachers at universities is mainly sourced from secondary sources and should not be treated as documentary evidence. We can only trust the sources we have and Biagioli’s views are feasible if the findings of specialists on Renaissance education such as Oskar Kristeller and Paul Grendler are set aside. If we do so, we can argue that abbaco topics were taught at the University of Perugia but that requires that we also set aside the work of mathematical historians, such as Jens Høyrup and Albrecht Heeffer, and that is, to me, a step too far.

The combination of what we can read in the extant commune records and from these scholars of education in the Renaissance tells us that men without degrees were not allowed to teach in university, that university was not the level at

⁴⁸ Boncompagni, 1879, *ibid.*, p. 437.

⁴⁹ Biagioli, M., *The Social Status of Italian Mathematicians 1450-1600, History of Science*, 1989, 27, pp. 41-95.

⁵⁰ Grendler, 1989, *ibid.*, footnote 40, pp. 419-420.

which a maestro d'abbaco taught, and that Pacioli was appointed as a maestro d'abbaco in all his appointments to teach in Perugia.

Upon leaving Perugia some time after April 1488, he went to Rome and obtained a public lectureship⁵¹ at the university.⁵² His next teaching appointment was in Naples⁵³ where he was appointed as a public lecturer at the university, probably for some time during the period between July 1489 and October 1490, perhaps also in 1491.⁵⁴

In 1491, while in Sansepolcro, Pacioli was instructed by his order to close the school for secular children, presumably an abbaco school. He may have next taught in the University of Padova in 1493⁵⁵ while, from dates in *De Scripturis* (his treatise on bookkeeping) he was working on his manuscript for *Summa Arithmetica*.

Following the publication of *Summa Arithmetica* in November 1494, his next known move was to Milan where he taught as a public lecturer in mathematics between 1496 and 1499 in the Scuole Palatine,⁵⁶ the most prestigious higher school in Milan. However, he was listed on the roll of the University of Pavia and his appointment in Milan was for the university.⁵⁷

His next known teaching appointment was for the University of Pisa, which was located temporarily in Florence between November 1500 and the end of October 1506.⁵⁸ The municipal records in Florence for appointments between 1500 and 1504 indicate that Pacioli, teacher of mathematics, was employed to teach Euclid.⁵⁹ During 1501-2, he was also listed as a teacher of mathematics at the University of Bologna but there is doubt that he ever took up the appointment,⁶⁰ though having two concurrent appointments would not have been impossible. In 1508, he was teaching in Venice⁶¹ at the Scuola di Rialto,⁶² though Ross⁶³ does not

⁵¹ Baldi, 1589, *ibid.*, carta 181r.

⁵² Grendler, 2002, *ibid.*, pp. 59-60; Taylor, 1942, *ibid.*, p. 161.

⁵³ Antinori, 2000, *ibid.*, p. 13.

⁵⁴ Ulivi (2009, p. 36) suggests that Pacioli was in Naples either between April 1488 and April 1489 or between July 1489 and October 1490, but the earlier date is not possible if he was working in Rome between 1488 and 1489.

⁵⁵ Antinori, 2000, *ibid.* p. 13.

⁵⁶ Ulivi, 2009, *ibid.*, p. 38.

⁵⁷ Grendler, 2002, *ibid.*, p. 87.

⁵⁸ Antinori, 2000, *ibid.*, p. 14

⁵⁹ Boncompagni, 1879, *ibid.*, p. 438.

⁶⁰ Antinori, 2000, *ibid.*, p. 14

⁶¹ Bagni, G. T., *Luca Pacioli, the mathematics of his time and De viribus quantitatis*. Forward accompanying Luca Pacioli's *De Viribus Quantitatis*. Petrucci: Città di Castello, pp. 21-28, 2009, p. 21.

⁶² Ceci, F., Fratini, F., Marioli, G., Pecchia, M., and Ricci, G., Luca Pacioli e la figura del mercante all'alba dell'età moderna con particolare riferimento a Venezia. In Martelli, M. (ed.) *Pacioli Fra Arte e Geometria*. Sansepolcro: Centro Studi "Mario Pancrazi", 2010, pp. 73-89 (p. 78).

⁶³ Ross (1976), *ibid.*

list Pacioli among those identified from the available records as having done so, suggesting that these were invited guest lectures.

It is not known where Pacioli spent all the years of his life but, of those periods for which details have survived and based upon the analysis above, where and when and at what level Pacioli taught is shown in Table 1. That there are gaps in the dates is entirely understandable. Pacioli was a senior member of his religious order and had many duties to perform in that role that would have required his full-time attention. He would also have spent considerable time gathering the material he included in *Summa Arithmetica* during the 1480s and early 1490s and for the publication of a number of titles in 1509.

Table 1: Where and at what level Pacioli taught⁶⁴

When	Where	Level
1464-70	Venice	Abbaco
1472-74	Naples	Abbaco
1475-80	Perugia	Abbaco
1486-88	Perugia	Abbaco
1488-89	Rome	University
1489-91	Naples	University
1491	Sansepolcro	Abbaco
1493	Padova	University
1496-99	Milan	University
1500-06	Florence ⁶⁵	University
1501-02	Bologna ⁶⁶	University
1507-08	Venice	University
1510-11	Perugia	Abbaco
1514	Rome	University

Thus, during his career, Pacioli worked in six of the 12 Italian universities, may have worked in another (Perugia) and also did some teaching at the ‘incomplete’ university at the Scuola di Rialto in Venice. He taught at the abbaco level in Venice, Naples, Perugia, and Sansepolcro. To the latter could be added his tuition of

⁶⁴ An earlier version of this table appeared in Sangster, A. and Scataglinibelghitar, G., Luca Pacioli: The Father of Accounting Education, *Accounting Education, an international journal*, 2010, Vol. 19, No. 4, pp. 423-438.

⁶⁵ Pacioli taught in Florence during the period it was the temporary location of the University of Pisa.

⁶⁶ Pacioli may never have taken up this appointment: Antinori, 2000, *ibid.* p. 14.

Leonardo da Vinci in Milan, though he clearly took Leonardo well beyond the level of *abbaco* instruction; and tuition of Albrecht Durer. Given his love of teaching and of spreading awareness and understanding of the relevance of mathematics to everything, it is likely that he taught many others of whom we have no details.

Pacioli's experience of teaching at both school and university level gave him a unique insight into the manner in which people of all ages learn and led to his identifying a better way to teach which he applied in his teaching of arithmetic and algebra and, in a similarly revolutionary manner, to his teaching of double entry bookkeeping.

Pacioli, the innovative compiler and teacher

For 200 years after Fibonacci's *Liber abaci*, *abbaco* texts were, typically, quite brief. However, as Elisabetta Ulivi describes, in the 15th century:

*"Manuscripts began to appear that, in scope and content, are placed at a much higher level than other abacus texts. These works... are considered to be large compendia of mathematical knowledge of the time. They cover, in a systematically organised manner, all subjects typical of the mathematics of the abacus, with the addition of questions often completely absent or only partially present in the shorter abacus texts."*⁶⁷

Ulivi refers to the work of Benedetto de Firenze (1429-79), spread across two separate manuscripts: *Pratica d'arismetricha* and *Pratica di geometria*. However, the indisputedly largest compendium of this type was Pacioli's *Summa Arithmetica* of 1494 – the manuscript for *Summa Arithmetica* was about five times the size of *Pratica di geometria*⁶⁸ – and, in the words of Maccagni and Giusti, not only was it a large book, it *"was an all encompassing work which summarised and rendered obsolete everything previously written about abbaco."*⁶⁹

Some view *Summa Arithmetica* as rather more than simply an *abbaco* teaching text and reference manual for merchants. Albrecht Heffer believes that, in the case of algebra, because of the innovative manner in which he dealt with the subject, revolutionised the way in which it was taught thereafter and, *"raised the testimonies*

⁶⁷ Ulivi, 2002, *ibid*, translation by the author.

⁶⁸ Heffer, A., Problem Solving to Argumentation: Pacioli's Appropriation of Abacus Algebra. *Foundations of Science*, 2008, Vol.13, No. 2, pp. 149-161.

⁶⁹ Maccagni, C. and Giusti, E.. *Luca Pacioli e la matematica del Rinascimento*. Firenze: Giunti, 1994, p. 18.

*of algebraic problem solving from the abacus masters to the next level of scientific discourse, the textbook...⁷⁰ ...”Pacioli’s appropriation of abacus texts in his *Summa* initiated an important restructuring of algebraic derivations into a theoretical introduction and its application in problem solving.⁷¹*

The transformation of Pacioli, the educator

What other abaco texts, including Pacioli’s Perugia manuscript of 1478, had all done was to present each problem as if it were one of a kind and then solve it using algebra. There was no immediate sense of generalisation. By the time he wrote *Summa Arithmetica*, Pacioli had come to appreciate that generalisation was not only possible, it was better.⁷²

In Distinctio 6 of *Summa Arithmetica*, Pacioli abandons the traditional abaco approach of presenting problems solved using algebra and, instead, presents a series of general propositions (or, ‘keys’) covering problems on proportions and partitions, demonstrates each one with an example, mainly taken from other abaco texts. His ‘keys’ are the theorems of Euclid and he states that such problems can be solved either using algebra or using a theorem of Euclid. He does not use algebra at that point to prove these keys but waits until he is discussing algebra in Distinctio 8. Before doing so, in Distinctio 7 he lists his ‘keys’.⁷³

By simply providing general principles upon which to make calculations of proportions and partitions, he avoided the need to introduce algebraic concepts at that level of education. As Heffer puts it, this “*was a completely new concept for the abacus tradition. We would argue that the idea is so essential to the rhetoric of a textbook that its first appearance in the *Summa* by Pacioli made... this early printed work the first textbook on algebra.*”⁷⁴

⁷⁰ Heffer, A., *The Rhetoric of Problems in Algebra Textbooks from Pacioli to Euler*. Preprint. Ghent University Centre for Logic and Philosophy of Science, Ghent, 2005, pp. 18-19. [Accessed on 4 June 2011 at <http://logica.ugent.be/albrecht/thesis/AlgebraRhetoric.pdf>]

⁷¹ Heffer, 2005, *ibid.*, p. 35.

⁷² Heffer, A., 2010a [From the second unknown to the symbolic equation In: Heffer, A. and Van Dyck, M., (eds.), *Philosophical Aspects of Symbolic Reasoning in Early-Modern Mathematics*, London: College Publications, pp. 57-102] attributes this to Pacioli’s knowledge of the *Trattato di Fioretti* of Antonio de’ Mazzinghi (c. 1380), an abaco tutor and expert in perspective who worked in Florence at the Scuola di Santa Trinita [Camerota, 2006, *ibid.*, p. 324]

⁷³ Heffer, A., Algebraic partitioning problems from Luca Pacioli’s Perugia manuscript (Vat. Lat. 3129), Sources and Commentaries in Exact Sciences, 2010b, Vol. 11, pp. 3–52 (pp. 19-20).

⁷⁴ Heffer, 2008, *ibid.*, p. 152.

Pacioli knew how to teach, knew what to teach, and knew how to present his teaching on a page of text in a manner that is not only accessible through his use of language but, also, enlightens students of all levels of experience and expertise. In doing so, he gave students of *abbaco* the principles of algebra in the classroom where previously they had simply learnt how to solve a range of seemingly unconnected problems. *Thereby, he shifted their knowledge from reliance on examples to reliance on generalizable principles.*

Those who subsequently consulted his *Summa Arithmetica* or the *abbaco* texts of his predecessors could then amplify their understanding through a study of the algebra, but students who experienced Pacioli's method in the classroom were far better equipped to deal with situations where knowledge of algebraic principles was useful. In effect, in true humanist educational spirit, he brought a level of understanding to everyone that had previously been, paraphrasing Heeffer,⁷⁵ "a trade secret passed to the few."

Summa Arithmetica demonstrates that Pacioli taught mathematics in this way and he would have had sufficient classroom experience to know the approach he adopted worked. But, in the case of *De Scripturis*, he adopted a similar approach despite there being *no* evidence that he *ever* taught the subject of double entry bookkeeping.

De Scripturis, exemplifies what we now consider to be the desirable attributes of a text designed to instruct.⁷⁶ Within it, he presented instruction in double entry bookkeeping in a manner that has not been repeated since, in which he learnt from the development in his approach to teaching algebra to focus upon learning from principles and so removed numbers from the instruction of double entry bookkeeping as much as possible so as not to interfere with the learning (also from principles),⁷⁷ an approach that has been found to be highly effective in a modern classroom.⁷⁸ It is unfortunate for generations of students of accounting that his unique generalisable-principles-based approach to the teaching of double entry bookkeeping was overlooked, leaving its teaching for hundreds of years to those who believed in the benefits of teaching from examples rather than teaching from

⁷⁵ Heeffer, A., 2011, In correspondence with the author.

⁷⁶ Sangster, A., Stoner, G., and McCarthy, P., Lessons for the classroom from Luca Pacioli, *Issues in Accounting Education*, 2008, Vol. 22(3): 447-457.

⁷⁷ Sangster, A. and Scataglinibelghitar, 2010, *ibid*.

⁷⁸ Sangster, A., Franklin, E., Abdul-Rahim, J., and Alwis, D., Using Pacioli's medieval text in today's introductory accounting course. Paper presented at the British Accounting Association Special Interest Group in Accounting Education annual conference, Dublin, Ireland, 2010, 26-28 May.

generalisable principles. In sum, Pacioli opened our eyes to the fact that problem solving is so much easier once we know the underlying principles to adopt in order to do so.

Notwithstanding his access to *abbaco* texts, it is probably fair to say that without his extensive experience at all levels of instruction, Pacioli may never have developed such insight into the manner in which people of all ages learn and led to his identifying a better way to teach which he applied in his teaching of arithmetic and algebra and, in a similarly revolutionary manner, to his teaching of double entry bookkeeping. The legacy of Luca Pacioli, the school teacher and the university professor, and his dedication to his calling as a teacher is, quite simply, beyond our ability to quantify, no matter how good we are at applying all that he and his successors have taught us.